

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Regular Expressions and Finite-State Automata

L445/L545

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Finite-state technology is:

- ▶ Fast and efficient
- ▶ Useful for a variety of language tasks

Three main topics we'll discuss:

- ▶ Regular Expressions (REs)
- ▶ Finite-State Automata (FSAs)
- ▶ Properties of Regular Languages

REs and FSAs are mathematically equivalent, but help us approach problems in different ways

Motivation

Regular  
expressions

RE syntax  
Additional functions

Regular languages

Properties

FSAs

Examples  
FSTNs  
Transition tables  
Determinism

FSTs

# Some useful tasks involving language

- ▶ Find all phone numbers in a text, e.g., occurrences such as

*When you call (614) 292-8833, you reach the fax machine.*

- ▶ Find multiple adjacent occurrences of the same word in a text, as in

*I read the the book.*

- ▶ Determine the language of the following utterance: French or Polish?

*Czy pasazer jadacy do Warszawy moze jechac przez Londyn?*

## Motivation

### Regular expressions

RE syntax

Additional functions

### Regular languages

Properties

### FSAs

Examples

FSTNs

Transition tables

Determinism

### FSTs

# More useful tasks involving language

- ▶ Look up the following words in a dictionary:  
*laughs, became, unidentifiable, Thatcherization*
- ▶ Determine the part-of-speech of words like the following, even if you can't find them in the dictionary:

*conurbation, cadence, disproportionality, lyricism,  
parlance*

Such tasks can be addressed using so-called finite-state machines (FSMs)

- ▶ We can start to understand FSMs by starting with the practical: regular expressions

## Motivation

### Regular expressions

RE syntax

Additional functions

### Regular languages

Properties

### FSAs

Examples

FSTNs

Transition tables

Determinism

### FSTs

# Regular expressions

A regular expression is a description of a set of strings, i.e., a language.

- ▶ They can be used to search for occurrences of these strings
  - ▶ A variety of unix tools (grep, sed), editors (emacs), and programming languages (perl, python) incorporate regular expressions.
- ▶ Just like any other formalism, regular expressions as such have no linguistic contents, but they can be used to refer to linguistic units.

# The syntax of regular expressions (1)

Regular expressions consist of

- ▶ strings of characters: `c`, `A100`, `natural language`, `30 years!`
- ▶ disjunction:
  - ▶ ordinary disjunction: `devoured|ate`, `famil(y|ies)`
  - ▶ character classes: `[Tt]he`, `bec[oa]me`
  - ▶ ranges: `[A-Z]` (a capital letter)
- ▶ negation:
  - ▶ `[^a]` (any symbol but `a`)
  - ▶ `[^A-Z0-9]` (not an uppercase letter or number)

# The syntax of regular expressions (2)

- ▶ counters

- ▶ optionality: ?

`colou?r`

- ▶ any number of occurrences: \* (Kleene star)

`[0-9]* years`

- ▶ at least one occurrence: +

`[0-9]+ dollars`

- ▶ wildcard for any character: .

`beg.n` for any character in between `beg` and `n`

- ▶ Parentheses to group items together

`ant(farm)?`

- ▶ Escaped characters to specify characters with special meanings:

`\*, \+, \?, \(, \), \|, \[, \]`

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# The syntax of regular expressions (3)

Operator precedence, from highest to lowest:

parentheses ( )

counters \* + ?

character sequences

disjunction |

- ▶  $\text{fire|ing} = \text{fire or ing}$
- ▶  $\text{fir}(e|\text{ing}) = \text{fir followed by either } e \text{ or ing}$

Note: The various unix tools and languages differ w.r.t. the exact syntax of the regular expressions they allow.



# Additional functionality for some RE uses (1)

Although not a part of our discussion about regular languages, some tools allow for more functionality

Anchors: anchor expressions to various parts of the string

- ▶  $\wedge$  = start of line
  - ▶ do not confuse with  $[\wedge \dots]$  used to express negation
- ▶  $\$$  = end of line
- ▶  $\backslash b$  non-word character
  - ▶ word characters are digits, underscores, or letters, i.e.,  $[0-9A-Za-z_]$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Additional functionality for some RE uses (2)

Use **aliases** to designate particular recurrent sets of characters

- ▶  $\backslash d = [0-9]$ : digit
- ▶  $\backslash D = [^\backslash d]$ : non-digit
- ▶  $\backslash w = [a-zA-Z0-9_]$ : alphanumeric
- ▶  $\backslash W = [^\backslash w]$ : non-alphanumeric
- ▶  $\backslash s = [\backslash r\backslash t\backslash n\backslash f]$ : whitespace character
  - ▶  $\backslash r$ : space,  $\backslash t$ : tab,  $\backslash n$ : newline,  $\backslash f$ : formfeed
- ▶  $\backslash S [^\backslash s]$ : non-whitespace

# Some RE practice

- ▶ What does  $\backslash \$ [0-9]^+ (\backslash . [0-9] [0-9])$  signify?
- ▶ Write a RE to capture the times on a digital watch (hours and minutes). Think about:
  - ▶ the (im)possible values for the hours
  - ▶ the (im)possible values for the minutes

We will view any formal **language** as a set of strings

- ▶ The language uses a finite vocabulary  $\Sigma$  (called an alphabet), and a set of string-combining **operations**
- ▶ Regular languages are the simplest class of formal languages
  - = class of languages definable by REs
  - = class of languages characterizable by FSAs

# Regular languages

How can the class of regular languages (specified by regular expressions) be characterized?

Let  $\Sigma$  be the set of all symbols of the language, the alphabet, then:

1.  $\{\}$  is a regular language
2.  $\forall a \in \Sigma: \{a\}$  is a regular language
3. If  $L_1$  and  $L_2$  are regular languages, so are:
  - 3.1 the concatenation of  $L_1$  and  $L_2$ :  
 $L_1 \cdot L_2 = \{xy \mid x \in L_1, y \in L_2\}$
  - 3.2 the union of  $L_1$  and  $L_2$ :  $L_1 \cup L_2$
  - 3.3 the Kleene closure of  $L$ :  $L^* = L_0 \cup L_1 \cup L_2 \cup \dots$  where  $L_i$  (roughly) represents  $L$  concatenated with itself  $i$  times, i.e.,

$$L_0 = \{\epsilon\}, L_{i+1} = \{ab \mid a \in L_i \text{ and } b \in L\}$$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Properties of regular languages (1)

The regular languages are closed under the following (where  $L_1$  and  $L_2$  are regular languages):

- ▶ concatenation:  $L_1 \cdot L_2$   
set of strings with beginning in  $L_1$  & continuation in  $L_2$
- ▶ Kleene closure:  $L_1^*$   
set of repeated concatenation of a string in  $L_1$
- ▶ union:  $L_1 \cup L_2$   
set of strings in  $L_1$  or in  $L_2$
- ▶ complementation:  $\Sigma^* - L_1$   
set of all possible strings that are not in  $L_1$

# Properties of regular languages (2)

The regular languages are closed under the following (where  $L_1$  and  $L_2$  are regular languages):

- ▶ difference:  $L_1 - L_2$   
set of strings which are in  $L_1$  but not in  $L_2$
- ▶ intersection:  $L_1 \cap L_2$   
set of strings in both  $L_1$  and  $L_2$
- ▶ reversal:  $L_1^R$   
set of the reversal of all strings in  $L_1$

# What sorts of expressions aren't regular?

In natural language, examples include **center-embedding** constructions.

- ▶ These dependencies are not regular:

- (1) a. The cat loves Mozart.  
b. The cat the dog chased loves Mozart.  
c. The cat the dog the rat bit chased loves Mozart.  
d. The cat the dog the rat the elephant admired bit chased loves Mozart.
- (2) (the noun)<sup>n</sup> (transitive-verb)<sup>n-1</sup> loves Mozart

- ▶ Similar ones would be regular:

- (3) A\*B\* loves Mozart

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

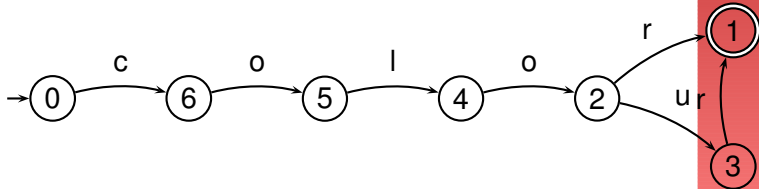


# Finite state machines

**Finite state machines** (or **automata**) (FSM, FSA) recognize or generate regular languages, exactly those specified by regular expressions.

Example:

- ▶ Regular expression: `colou?r`
- ▶ Finite state machine (representation):



Motivation

Regular expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Accepting/Rejecting strings

The behavior of an FSA is completely determined by its transition table.

- ▶ Assumption: there is a tape, with the input symbols read off consecutive cells of the tape
  - ▶ The machine starts in the start (initial) state, about to read the contents of the first cell on the input tape
  - ▶ The FSA uses the transition table to decide where to go at each step
- ▶ A string is rejected in exactly two cases:
  1. A transition on an input symbol takes you nowhere
  2. The state you're in after processing the entire input is not an accept (final) state
- ▶ Otherwise, the string is accepted.

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

# Defining finite state automata

A **finite state automaton** is a quintuple  $(Q, \Sigma, E, S, F)$  with

- ▶  $Q$  a finite set of states
- ▶  $\Sigma$  a finite set of symbols, the alphabet
- ▶  $S \subseteq Q$  the set of start states
- ▶  $F \subseteq Q$  the set of final states
- ▶  $E$  a set of edges  $Q \times (\Sigma \cup \{\epsilon\}) \times Q$

The **transition function**  $d$  can be defined as:

$$d(q, a) = \{q' \in Q \mid \exists (q, a, q') \in E\}$$

# Example FSA

FSA to recognize strings of the form:  $[ab]^+$

- ▶ i.e.,  $L = \{ a, b, ab, ba, aab, bab, aba, bba, \dots \}$

FSA is defined as:

- ▶  $Q = \{0, 1\}$
- ▶  $\Sigma = \{a, b\}$
- ▶  $S = \{0\}$
- ▶  $F = \{1\}$
- ▶  $E = \{(0, a, 1), (0, b, 1), (1, a, 1), (1, b, 1)\}$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

# FSA: set of zero or more $a$ 's

$$L = \{ \epsilon, a, aa, aaa, aaaa, \dots \}$$

- ▶  $Q = \{0\}$
- ▶  $\Sigma = \{a\}$
- ▶  $S = \{0\}$
- ▶  $F = \{0\}$
- ▶  $E = \{(0, a, 0)\}$

# FSA: set of all lowercase alphabetic strings ending in $b$

L captured by  $[a-z]^*b$

$$= \{b, ab, tb, \dots, aab, abb, \dots\}$$

- ▶  $Q = \{0, 1\}$
- ▶  $\Sigma = \{a, b, c, \dots, z\}$
- ▶  $S = \{0\}$
- ▶  $F = \{1\}$
- ▶  $E = \{(0, a, 0), (0, c, 0), (0, d, 0), \dots, (0, z, 0), (0, b, 1), (1, b, 1), (1, a, 0), (1, c, 0), (1, d, 0), \dots (1, z, 0)\}$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# FSA: the set of all strings in $[ab]^*$ with exactly 2 $a$ 's

Do this yourself

It might help to first rewrite a more precise regular expression for this

- ▶ First, be clear what the domain is (all strings in  $[ab]^*$ )
- ▶ And then figure out how to narrow it down

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

# Language accepted by an FSA

The extended set of edges  $\hat{E} \subseteq Q \times \Sigma^* \times Q$  is the smallest set such that

- ▶  $\forall (q, \sigma, q') \in E : (q, \sigma, q') \in \hat{E}$
- ▶  $\forall (q_0, \sigma_1, q_1), (q_1, \sigma_2, q_2) \in \hat{E} : (q_0, \sigma_1\sigma_2, q_2) \in \hat{E}$

The **language  $L(A)$  of a finite state automaton  $A$**  is defined as:

$$L(A) = \{w \mid q_s \in S, q_f \in F, (q_s, w, q_f) \in \hat{E}\}$$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's



# FSA for simple NPs

Where  $d$  is an alias for determiners,  $a$  for adjectives, and  $n$  for nouns:

- ▶  $Q = \{0, 1, 2\}$
- ▶  $\Sigma = \{d, a, n\}$
- ▶  $S = \{0\}$
- ▶  $F = \{2\}$
- ▶  $E = \{(0, d, 1), (0, \epsilon, 1), (1, a, 1), (1, n, 2), (2, n, 2)\}$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FST's

Transition tables

Determinism

FST's

# Finite state transition networks (FSTN)

Finite state transition networks are graphical descriptions of finite state machines:

- ▶ Nodes represent the states
  - ▶ Start states are marked with a short arrow
  - ▶ Final states are indicated by a double circle
- ▶ Arcs represent the transitions

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

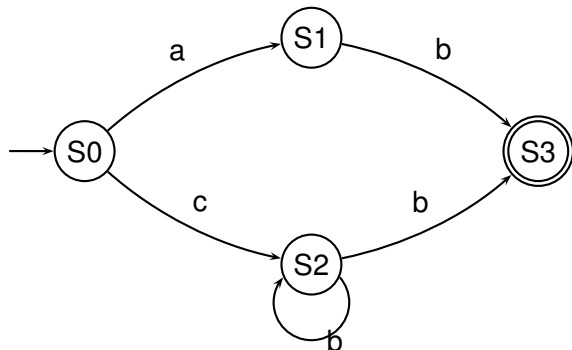
FSTN's

Transition tables

Determinism

FST's

# Example for a finite state transition network



Regular expression specifying the language generated or accepted by the corresponding FSM:  $ab \mid cb^+$

Motivation

Regular  
expressions

RE syntax  
Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Finite state transition tables

Finite state transition tables are an alternative, textual way of describing finite state machines:

- ▶ Rows represent the states
  - ▶ Start states are marked with a dot after their name
  - ▶ Final states with a colon
- ▶ Columns represent the alphabet
- ▶ Fields in the table encode the transitions

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# The example specified as finite state transition table

	a	b	c	d
S0.	S1		S2	
S1		S3:		
S2		S2,S3:		
S3:				

# Some properties of finite state machines

- ▶ Recognition problem can be solved in linear time (independent of the size of the automaton).
- ▶ There is an algorithm to transform each automaton into a unique equivalent automaton with the least number of states.

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

# Deterministic Finite State Automata

A finite state automaton is deterministic iff it has

- ▶ no  $\epsilon$  transitions and
- ▶ for each state and each symbol there is at most one applicable transition.

Every non-deterministic automaton can be transformed into a deterministic one:

- ▶ Define new states representing a disjunction of old states for each non-determinacy which arises.
- ▶ Define arcs for these states corresponding to each transition which is defined in the non-deterministic automaton for one of the disjuncts in the new state names.

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

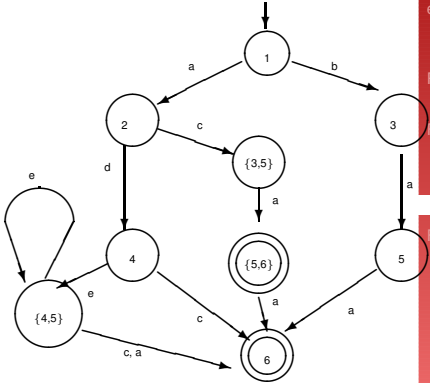
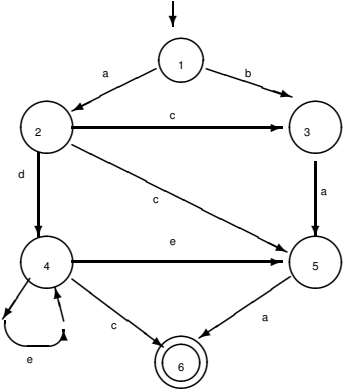
FSTNs

Transition tables

Determinism

FSTs

# Example: Determinization of FSA



Motivation

Regular expressions

- RE syntax
- Additional functions

Regular languages

- Properties

FSA's

- Examples
- FSTNs
- Transition tables

Determinism

FSTs



# Why finite-state?

## *Finite* number of states

- ▶ Number of states bounded in advance – determined by its transition table
- ▶ Therefore, the machine has a limit to the amount of memory it uses.
  - ▶ Its behavior at each stage is based on the transition table, and depends just on the state it's in & the input.
  - ▶ So, the current state reflects the history of the processing so far.

Classes of formal languages which are not regular require additional memory to keep track of previous information, e.g., center-embedding constructions

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's

Needed: mechanism to keep track of path taken

A **finite state transducer** is a 6-tuple  $(Q, \Sigma_1, \Sigma_2, E, S, F)$  with

- ▶  $Q$  a finite set of states
- ▶  $\Sigma_1$  a finite set of symbols, the input alphabet
- ▶  $\Sigma_2$  a finite set of symbols, the output alphabet
- ▶  $S \subseteq Q$  the set of start states
- ▶  $F \subseteq Q$  the set of final states
- ▶  $E$  a set of edges  $Q \times (\Sigma_1 \cup \{\epsilon\}) \times Q \times (\Sigma_2 \cup \{\epsilon\})$

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

Examples

FSTNs

Transition tables

Determinism

FSTs

# Transducers and determinization

A finite state transducer understood as consuming an input and producing an output cannot generally be determinized.

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSAs

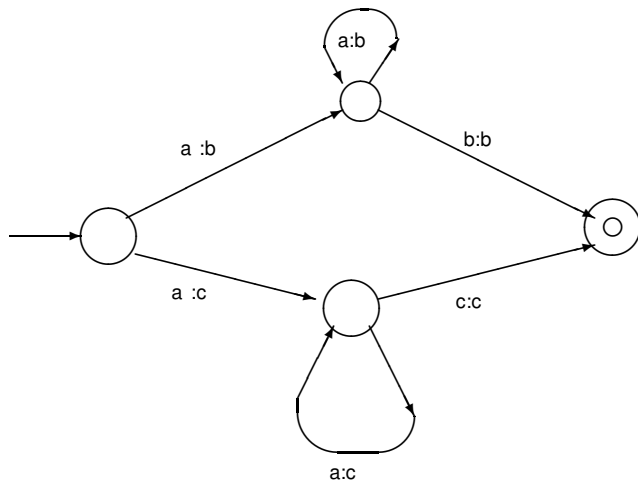
Examples

FSTNs

Transition tables

Determinism

FSTs



- ▶ Notations for characterizing regular languages:
  - ▶ Regular expressions
  - ▶ Finite state transition networks
  - ▶ Finite state transition tables
- ▶ Finite state machines and regular languages:  
Definitions and some properties
- ▶ Finite state transducers (more later)

Motivation

Regular  
expressions

RE syntax

Additional functions

Regular languages

Properties

FSA's

Examples

FSTN's

Transition tables

Determinism

FST's