Chiastic Lambda-Calculi

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Examples and Motivation

Associative λ-calculi

Chiastic λ-calculi

$\langle\langle\lambda_\to\rangle\rangle_{\chi L}$ in action

Examples

Scrambling in Japanese

- *Tarou -ga* *hon -wo* *yon-da*
  — NOM book ACC read-PERF

- *hon -wo* *Tarou -ga* *yon-da*
  book ACC — NOM read-PERF
  'Taro read the book.'

Keyword arguments

- `yonda(wo='hon', ga='Tarou')`
- `yonda(ga='Tarou', wo='hon')`

Shorthands in category theory

- FG where $(FG)(X) = F(GX)$
- ηF where $(\eta F)(X) = \eta_{FX}$
- $F\eta$ where $(F\eta)(X) = F(\eta_X)$

What do these have in common?

Juxtaposition is (essentially) associative

$$(f\,g)\,x \;\approx\; f\,(g\,x)$$

Application is (essentially) commutative

$$f\,x\,y \;\approx\; f\,y\,x$$

Our Goal convert those "$\approx$" into "$=$"

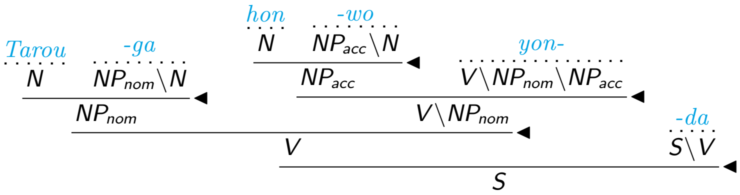
Scrambling in Japanese

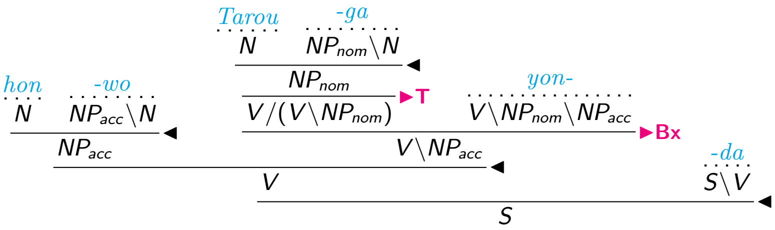
Many languages have "free word order"

- *Tarou -ga* *hon -wo* *yon-da*
  — NOM book ACC read-PERF
- *hon -wo* *Tarou -ga* *yon-da*
  book ACC — NOM read-PERF
  'Taro read the book.'

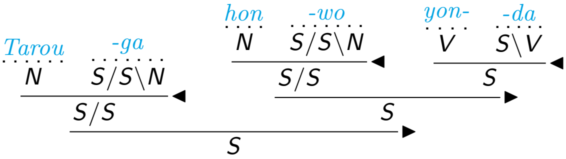
  - Both orders are normal and natural
  - Both have the same propositional content
  - Though, information structure may differ

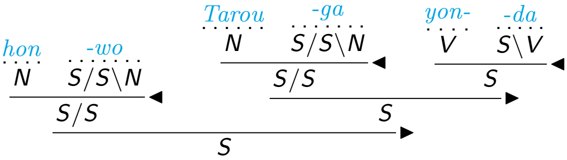
Arguments: Chomskian-style accounts





Adjuncts: Radical neo-Davidsonian accounts





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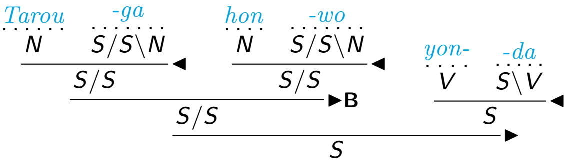

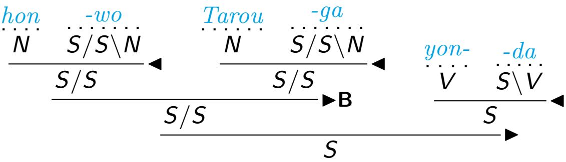

Arguments vs Adjuncts

Why prefer adjuncts?

- Avoids the need for **T** and **Bx** (they're dangerous together)
- Syntax matches morphology/prosody
- Same parse tree for different word orders (commutativity)
- Online and partial parsing is easy (associativity)

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Only moves the problem from syntax to semantics!

- Also true of other CCG approaches to scrambling

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Chiastic λ-calculi solve the problem
(in the semantics)

What are functions?

Traditional λ-calculi intentionally confuse two ideas

- **Procedures** operations mapping values to values
- **Data** values representing procedures

Category theory keeps them distinct

- **Morphisms** functions as procedures
- **Exponentials** functions as data

For associativity, we must keep them distinct too

- $(\lambda x.\ e)$ Unbracketed abstractions are procedures
- $\langle\!\langle \lambda x.\ e \rangle\!\rangle$ Bracketed abstractions are values

Associative λ-calculi: $\langle\!\langle \lambda \rangle\!\rangle$

Variables	$x, y, z, \ldots$			
Terms	$e, f, g, \ldots$	$::=$	x	*variables*
		$\mid$	$(\lambda x.\ e)$	*abstraction*
		$\mid$	$\langle\!\langle e \rangle\!\rangle$	*bracketing*
		$\mid$	$e \cdot f$	*juxtaposition*

$$\frac{}{(\lambda x.\ f) \cdot \langle\!\langle e \rangle\!\rangle \rightsquigarrow \{x \mapsto e\} f}\ \textsc{Beta}$$

$$\frac{}{(e \cdot f) \cdot g \equiv e \cdot (f \cdot g)}\ \textsc{Assoc}$$

What does juxtaposition mean?

Application $(\lambda x.\, e) \cdot \langle\!\langle f \rangle\!\rangle$

Composition $(\lambda x.\, e) \cdot (\lambda y.\, f)$

$$\Big((\lambda x.\, e) \cdot (\lambda y.\, f)\Big) \cdot \langle\!\langle g \rangle\!\rangle \equiv (\lambda x.\, e) \cdot \Big((\lambda y.\, f) \cdot \langle\!\langle g \rangle\!\rangle\Big)$$

Tupling $\langle\!\langle f \rangle\!\rangle \cdot \langle\!\langle g \rangle\!\rangle$

$$(\lambda x. \lambda y.\, e) \cdot \Big(\langle\!\langle f \rangle\!\rangle \cdot \langle\!\langle g \rangle\!\rangle\Big) \equiv \Big((\lambda x. \lambda y.\, e) \cdot \langle\!\langle f \rangle\!\rangle\Big) \cdot \langle\!\langle g \rangle\!\rangle$$

How powerful is it?

$\langle\!\langle \mathscr{L} \rangle\!\rangle$ is at least as powerful as $\mathscr{L}$

- Every $\mathscr{L}$-term has an evaluation-equivalent $\langle\!\langle \mathscr{L} \rangle\!\rangle$-term

$$
\begin{aligned}
[\![x]\!] &= x \\
[\![(\lambda x.\ e)]\!] &= (\lambda x.\ [\![e]\!]) \\
&\vdots \\
[\![e \cdot f]\!] &= [\![e]\!] \cdot \langle\!\langle [\![f]\!] \rangle\!\rangle \\
[\![(e)]\!] &= [\![e]\!]
\end{aligned}
$$

$\langle\!\langle \mathscr{L} \rangle\!\rangle$ can be more expressive than $\mathscr{L}$

- $\langle\!\langle \lambda_\to \rangle\!\rangle$ has tuples, but they can't be encoded in $\lambda_\to$
    - Then again, almost everything stronger than $\lambda_\to$ has tuples

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Chiastic λ-calculi

The term level

- Syntax — Two flavors of chiasmus
- Equivalence
- Reduction
- Sanity check

The type level

- Syntax
- Equivalence
- Reduction
- Sanity check — Well-formed types

Formalizing restricted commutativity

Actually we **don't** want full commutativity

- *Tarou -ga kuruma -ga ar-u*
  — NOM car NOM have-NPST
  'Taro has a car.'

% *kuruma -ga Tarou -ga ar-u*
  car NOM — NOM have-NPST
  'The car has a Taro.'

- Let a dimension denote a class of non-commuting elements
- Elements along different dimensions don't interfere with one another

Chiastic λ-calculi: $\langle\!\langle \lambda \rangle\!\rangle_\chi$

Variables	$x, y, z, \ldots$			
Dimensions	$A, B, C, \ldots$			
Terms	$e, f, g, \ldots$	$::=$	x	*variables*
		$\mid$	$(\lambda_A x.\ e)$	*abstraction*
		$\mid$	$\langle\!\langle e \rangle\!\rangle_A$	*bracketing*
		$\mid$	$e \cdot f$	*juxtaposition*

- Choose one

$$\frac{A \neq B}{(\lambda_A x.\ \lambda_B y.\ e) \ \equiv\ (\lambda_B y.\ \lambda_A x.\ e)}\ \textsc{Chi\_L}$$

$$\frac{A \neq B}{\langle\!\langle e \rangle\!\rangle_A \cdot \langle\!\langle f \rangle\!\rangle_B \ \equiv\ \langle\!\langle f \rangle\!\rangle_B \cdot \langle\!\langle e \rangle\!\rangle_A}\ \textsc{Chi\_R}$$

Chiastic λ-calculi: $\langle\!\langle \lambda \rangle\!\rangle_{\chi}$

Variables	$x, y, z, \ldots$			
Dimensions	$A, B, C, \ldots$			
Terms	$e, f, g, \ldots$	$::=$	x	*variables*
		$\mid$	$(\lambda_A x.\ e)$	*abstraction*
		$\mid$	$\langle\!\langle e \rangle\!\rangle_A$	*bracketing*
		$\mid$	$e \cdot f$	*juxtaposition*

- For this talk, we'll only consider CHI_L

$$\frac{A \neq B}{(\lambda_A x.\ \lambda_B y.\ e) \ \equiv\ (\lambda_B y.\ \lambda_A x.\ e)}\ \text{CHI\_L}$$

$$\frac{A \neq B}{\langle\!\langle e \rangle\!\rangle_A \cdot \langle\!\langle f \rangle\!\rangle_B \ \equiv\ \langle\!\langle f \rangle\!\rangle_B \cdot \langle\!\langle e \rangle\!\rangle_A}\ \text{CHI\_R}$$

CHI_L vs CHI_R

Should we accept terms like this?

$$(\lambda_A x.\ \lambda_B y.\ e) \cdot (\lambda_C z.\ \langle\!\langle a \rangle\!\rangle_A) \cdot \langle\!\langle b \rangle\!\rangle_B \cdot \langle\!\langle c \rangle\!\rangle_C$$

Should we accept sentences like this?

- [*sono hon -wo* ⟨*Hanako -ga*⟩ *Tarou -ga kat-ta*] *-to*
  that book ACC — NOM — NOM buy-PERF COMP
  omot-te iru
  think.PROG

  'Hanako thinks that Taro bought that book.'

Term equivalence for $\langle\!\langle\lambda\rangle\!\rangle_{\chi L}$

$$\boxed{e \;\equiv\; f}$$

$$\frac{A \neq B}{(\lambda_A x.\ \lambda_B y.\ e) \;\equiv\; (\lambda_B y.\ \lambda_A x.\ e)} \qquad \frac{}{e \cdot (f \cdot g) \;\equiv\; (e \cdot f) \cdot g}$$

$$\frac{e \;\equiv\; e'}{(\lambda_A x.\ e) \;\equiv\; (\lambda_A x.\ e')} \qquad \frac{e \;\equiv\; e'}{\langle\!\langle e \rangle\!\rangle_A \;\equiv\; \langle\!\langle e' \rangle\!\rangle_A} \qquad \frac{e \;\equiv\; e' \qquad f \;\equiv\; f'}{e \cdot f \;\equiv\; e' \cdot f'}$$

$$\frac{}{e \;\equiv\; e} \qquad \frac{f \;\equiv\; e}{e \;\equiv\; f} \qquad \frac{e \;\equiv\; f \qquad f \;\equiv\; g}{e \;\equiv\; g}$$

Term reduction for $\langle\!\langle\lambda\rangle\!\rangle_{\chi L}$

$\boxed{e \rightsquigarrow e'}$

$$\frac{h \equiv (\lambda_A x.\, f)}{h \cdot \langle\!\langle e \rangle\!\rangle_A \rightsquigarrow \{x \mapsto e\} f} \qquad \frac{e \rightsquigarrow e'}{(\lambda_A x.\, e) \rightsquigarrow (\lambda_A x.\, e')}$$

$$\frac{e \rightsquigarrow e'}{\langle\!\langle e \rangle\!\rangle_A \rightsquigarrow \langle\!\langle e' \rangle\!\rangle_A} \qquad \frac{e \rightsquigarrow e'}{e \cdot f \rightsquigarrow e' \cdot f} \qquad \frac{f \rightsquigarrow f'}{e \cdot f \rightsquigarrow e \cdot f'}$$

$$\frac{e \cdot f \rightsquigarrow h}{e \cdot (f \cdot g) \rightsquigarrow h \cdot g} \qquad \frac{f \cdot g \rightsquigarrow h}{(e \cdot f) \cdot g \rightsquigarrow e \cdot h}$$

Do our terms make sense?

Theorem Term reduction is weak Church–Rosser.

Proof There are no critical pairs.

Corollary Term reduction is Church–Rosser.

Proof Supposing we can prove strong normalization, then just use Newman's lemma.

Conjecture Term reduction (for $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi L}$) is strongly normalizing.

Remark This is suspiciously difficult to prove.

What are types?

The intrinsic view (à la Church)

- Types are manifest in terms
- Terms can have only one type
- Ill-typed terms "don't exist"

The extrinsic view (à la Curry)

- Types characterize properties of terms
- Terms could have multiple types
- All terms exist, but we only care about the well-typed ones

What are types?

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Our view

- Types give abstract interpretations of terms

$$\frac{\Gamma \vdash e \triangleright \tau \qquad \tau \rightsquigarrow^* \tau'}{\exists e'.\quad e \rightsquigarrow^* e' \quad \wedge \quad \Gamma \vdash e' \triangleright \tau'}$$

Simply-typed left-chiastic λ-calculus: $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi_L}$

Types	$\sigma, \tau, \upsilon, \ldots$	$::=$	T	*primitive types*
		$\mid$	$\sigma \xrightarrow{A} \tau$	*arrow types*
		$\mid$	$\langle\!\langle \tau \rangle\!\rangle_A$	*bracketed types*
		$\mid$	$\sigma \cdot \tau$	*juxtaposition*

$\boxed{\Gamma \vdash e \rhd \tau}$

$$\frac{\vdash_{ctx} \Gamma \qquad \Gamma(x) \equiv \tau}{\Gamma \vdash x \rhd \tau} \qquad \frac{\Gamma, x{:}\sigma \vdash e \rhd \tau}{\Gamma \vdash (\lambda_A x.\, e) \rhd \sigma \xrightarrow{A} \tau}$$

$$\frac{\Gamma \vdash e \rhd \tau}{\Gamma \vdash \langle\!\langle e \rangle\!\rangle_A \rhd \langle\!\langle \tau \rangle\!\rangle_A} \qquad \frac{\Gamma \vdash e \rhd \sigma \qquad \Gamma \vdash f \rhd \tau}{\Gamma \vdash e{\cdot}f \rhd \sigma{\cdot}\tau}$$

Type equivalence for $\langle\!\langle \lambda_{\to} \rangle\!\rangle_{\chi L}$

$\boxed{\tau \equiv \sigma}$

$$\frac{A \neq B}{\sigma \xrightarrow{A} \tau \xrightarrow{B} \upsilon \equiv \tau \xrightarrow{B} \sigma \xrightarrow{A} \upsilon} \qquad \frac{}{\sigma \cdot (\tau \cdot \upsilon) \equiv (\sigma \cdot \tau) \cdot \upsilon}$$

$$\frac{\sigma \equiv \sigma' \qquad \tau \equiv \tau'}{\sigma \xrightarrow{A} \tau \equiv \sigma' \xrightarrow{A} \tau'} \qquad \frac{\tau \equiv \tau'}{\langle\!\langle \tau \rangle\!\rangle_A \equiv \langle\!\langle \tau' \rangle\!\rangle_A} \qquad \frac{\sigma \equiv \sigma' \qquad \tau \equiv \tau'}{\sigma \cdot \tau \equiv \sigma' \cdot \tau'}$$

$$\frac{}{\tau \equiv \tau} \qquad \frac{\sigma \equiv \tau}{\tau \equiv \sigma} \qquad \frac{\sigma \equiv \tau \qquad \tau \equiv \upsilon}{\sigma \equiv \upsilon}$$

Type reduction for $\langle\!\langle\lambda_{\rightarrow}\rangle\!\rangle_{\chi L}$

$\boxed{\tau \ \rightsquigarrow \ \tau'}$

$$\frac{\rho \ \equiv \ \sigma \xrightarrow{A} \tau}{\rho \cdot \langle\!\langle \sigma \rangle\!\rangle_A \ \rightsquigarrow \ \tau} \qquad \frac{\sigma \ \rightsquigarrow \ \sigma'}{\sigma \xrightarrow{A} \tau \ \rightsquigarrow \ \sigma' \xrightarrow{A} \tau} \qquad \frac{\tau \ \rightsquigarrow \ \tau'}{\sigma \xrightarrow{A} \tau \ \rightsquigarrow \ \sigma \xrightarrow{A} \tau'}$$

$$\frac{\tau \ \rightsquigarrow \ \tau'}{\langle\!\langle \tau \rangle\!\rangle_A \ \rightsquigarrow \ \langle\!\langle \tau' \rangle\!\rangle_A} \qquad \frac{\sigma \ \rightsquigarrow \ \sigma'}{\sigma \cdot \tau \ \rightsquigarrow \ \sigma' \cdot \tau} \qquad \frac{\tau \ \rightsquigarrow \ \tau'}{\sigma \cdot \tau \ \rightsquigarrow \ \sigma \cdot \tau'}$$

$$\frac{\sigma \cdot \tau \ \rightsquigarrow \ \rho}{\sigma \cdot (\tau \cdot \upsilon) \ \rightsquigarrow \ \rho \cdot \upsilon} \qquad \frac{\tau \cdot \upsilon \ \rightsquigarrow \ \rho}{(\sigma \cdot \tau) \cdot \upsilon \ \rightsquigarrow \ \tau \cdot \rho}$$

Do our types make sense?

Every type has a normal form

Theorem Type reduction for $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi L}$ is strongly normalizing Proof

Theorem Type reduction for $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi L}$ is Church–Rosser Proof

So we can define

$$\frac{\Gamma \vdash e \triangleright \tau_0 \qquad NF(\tau_0) \equiv \tau \qquad \vdash_{type} \tau}{\Gamma \vdash e : \tau}$$

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But, what does unresolved type juxtaposition mean?

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Good $\langle\langle\sigma\rangle\rangle_A \cdot \langle\langle\tau\rangle\rangle_B$

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Good $\langle\!\langle \sigma \rangle\!\rangle_A \cdot \langle\!\langle \tau \rangle\!\rangle_B$

Bad $(\sigma \xrightarrow{A} \langle\!\langle \tau \rangle\!\rangle_B) \cdot \langle\!\langle \upsilon \rangle\!\rangle_C$ where $A \neq C$

$(\sigma \xrightarrow{A} \tau) \cdot \langle\!\langle \upsilon \rangle\!\rangle_A$ where $\sigma \not\equiv \upsilon$

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$(\sigma \xrightarrow{A} \tau) \cdot \langle\!\langle \upsilon \rangle\!\rangle_A$ where $\sigma \not\equiv \upsilon$

Ugly $\langle\!\langle \sigma \rangle\!\rangle_A \cdot (\tau \xrightarrow{B} \upsilon)$

$(\rho \xrightarrow{A} \sigma) \cdot (\tau \xrightarrow{B} \upsilon)$

Do our types make sense?

Every type has a normal form

But, what does unresolved type juxtaposition mean?

Good $\langle\!\langle \sigma \rangle\!\rangle_A \cdot \langle\!\langle \tau \rangle\!\rangle_B$

Bad $(\sigma \xrightarrow{A} \langle\!\langle \tau \rangle\!\rangle_B) \cdot \langle\!\langle \upsilon \rangle\!\rangle_C$ where $A \neq C$

$(\sigma \xrightarrow{A} \tau) \cdot \langle\!\langle \upsilon \rangle\!\rangle_A$ where $\sigma \not\equiv \upsilon$

Ugly $\langle\!\langle \sigma \rangle\!\rangle_A \cdot (\tau \xrightarrow{B} \upsilon)$

$(\rho \xrightarrow{A} \sigma) \cdot (\tau \xrightarrow{B} \upsilon)$

- If $\vdash_{type} \tau$ doesn't accept ugly terms, then it doesn't have the subterm property.

Using $\langle\langle\lambda_{\rightarrow}\rangle\rangle_{\chi L}$ to describe Japanese

Noun phrase scrambling

Tarou-ga hon-wo yonda
Hon-wo Tarou-ga yonda

Verbal morphology

"Paradoxical" behavior
Resolving the paradox

Semantic analysis of *Tarou-ga*

Tarou · · · · · · · · · N

-ga · · · · · · · · · $S/S\backslash N$

——————————————— ◀

S/S

——————————————— β

S/S

Semantic analysis of *Tarou-ga*

$$\dfrac{\dfrac{\mathit{Tarou}}{\langle\!\langle \mathit{Taro}' \rangle\!\rangle_N : N} \qquad \dfrac{\mathit{\text{-}ga}}{(\lambda_N n.\, \lambda_S s.\, \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_S) : S/S\backslash N}}{\dfrac{(\lambda_N n.\, \lambda_S s.\, \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_N : S/S}{(\lambda_S s.\, \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S) : S/S}\,\beta}\,\blacktriangleleft$$

Semantic analysis of *Tarou-ga*

$$\dfrac{\dfrac{\textit{Tarou}}{\langle\!\langle \mathit{Taro}' \rangle\!\rangle_{N}} \qquad \dfrac{\textit{-ga}}{(\lambda_{N}\, n.\ \lambda_{S}\, s.\ \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_{S})}}{\dfrac{(\lambda_{N}\, n.\ \lambda_{S}\, s.\ \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_{S}) \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{N}}{(\lambda_{S}\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_{S})}\,\beta}\,\blacktriangleleft$$

Semantic analysis of *Tarou-ga*

$$\dfrac{\dfrac{\dfrac{\textit{Tarou}}{\langle\!\langle \mathit{Taro}' \rangle\!\rangle_N} \qquad \dfrac{\textit{-ga}}{(\lambda_N\, n.\ \lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_S)}}{(\lambda_N\, n.\ \lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle n \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_N}\blacktriangleleft}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S)}\beta$$

Semantic analysis of *Tarou-ga hon-wo yonda*

$$
\dfrac{\dfrac{\textit{Tarou-ga}}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S)} \quad \dfrac{\dfrac{\dfrac{\dfrac{\textit{hon-wo}}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S)} \quad \dfrac{\textit{yonda}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S}}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S} \blacktriangleright}{\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a) \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S}\beta}{\langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S}\beta}{\dfrac{\dfrac{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S}{\langle\!\langle (\lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}') \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S}\beta}{\langle\!\langle \mathit{Taro}'\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S}\beta} \blacktriangleright
$$

Semantic analysis of *Tarou-ga hon-wo yonda*

hon-wo

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S)$$

yonda

$$\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S$$

$$\blacktriangleright$$

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a) \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

Tarou-ga

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S)$$

$$\blacktriangleright$$

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle (\lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}') \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle \mathit{Taro}'\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

Semantic analysis of *Tarou-ga hon-wo yonda*

hon-wo

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S)$$

yonda

$$\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S$$

$$\frac{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \qquad \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S} \blacktriangleright$$

$$\frac{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S}{\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a) \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S} \beta$$

$$\frac{\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a) \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S}{\langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S} \beta$$

Tarou-ga

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S)$$

$$\frac{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \qquad \langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S}{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S} \blacktriangleright$$

$$\frac{(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S}{\langle\!\langle (\lambda_{nom}\, n.\ n\ read'\ book') \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S} \beta$$

$$\frac{\langle\!\langle (\lambda_{nom}\, n.\ n\ read'\ book') \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S}{\langle\!\langle Taro'\ read'\ book' \rangle\!\rangle_S} \beta$$

Semantic analysis of *Tarou-ga hon-wo yonda*

hon-wo

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S)$$

yonda

$$\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S$$

Combined ($\blacktriangleright$):

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S$$

β:

$$\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a) \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S$$

β:

$$\langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S$$

Tarou-ga

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S)$$

Combined ($\blacktriangleright$):

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S$$

β:

$$\langle\!\langle (\lambda_{nom}\, n.\ n\ read'\ book') \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S$$

β:

$$\langle\!\langle Taro'\ read'\ book' \rangle\!\rangle_S$$

Semantic analysis of *Tarou-ga hon-wo yonda*

hon-wo

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S)$$

yonda

$$\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S$$

$$\blacktriangleright$$

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ read'\ a) \cdot \langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S$$

$$\beta$$

Tarou-ga

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S)$$

$$\langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S$$

$$\blacktriangleright$$

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ read'\ book' \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle (\lambda_{nom}\, n.\ n\ read'\ book') \cdot \langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S$$

$$\beta$$

$$\langle\!\langle Taro'\ read'\ book' \rangle\!\rangle_S$$

Semantic analysis of *Tarou-ga hon-wo yonda*

hon-wo

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S)$$

yonda

$$\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S$$

$$\blacktriangleright\quad (\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \mathit{read}'\ a) \cdot \langle\!\langle \mathit{book}' \rangle\!\rangle_{acc} \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

Tarou-ga

$$(\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S)$$

$$\blacktriangleright\quad (\lambda_S\, s.\ \langle\!\langle s \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle (\lambda_{nom}\, n.\ n\ \mathit{read}'\ \mathit{book}') \cdot \langle\!\langle \mathit{Taro}' \rangle\!\rangle_{nom} \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle \mathit{Taro}'\ \mathit{read}'\ \mathit{book}' \rangle\!\rangle_S$$

Semantic analysis of *hon-wo Tarou-ga yonda*

$$\dfrac{\textit{Tarou-ga}}{(\lambda_S\, s.\ \langle\!\langle s\cdot\langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S)} \qquad \dfrac{\textit{yonda}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \ read'\ \ a \rangle\!\rangle_S}$$

$$\blacktriangleright\quad (\lambda_S\, s.\ \langle\!\langle s\cdot\langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S)\cdot\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \ read'\ \ a \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ n\ \ read'\ \ a)\cdot\langle\!\langle Taro' \rangle\!\rangle_{nom} \rangle\!\rangle_S$$

$$\beta(\chi_L)\quad \langle\!\langle \lambda_{acc}\, a.\ Taro'\ \ read'\ \ a \rangle\!\rangle_S$$

$$\dfrac{\textit{hon-wo}}{(\lambda_S\, s.\ \langle\!\langle s\cdot\langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S)}$$

$$\blacktriangleright\quad (\lambda_S\, s.\ \langle\!\langle s\cdot\langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S)\cdot\langle\!\langle \lambda_{acc}\, a.\ Taro'\ \ read'\ \ a \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle (\lambda_{acc}\, a.\ Taro'\ \ read'\ \ a)\cdot\langle\!\langle book' \rangle\!\rangle_{acc} \rangle\!\rangle_S$$

$$\beta\quad \langle\!\langle Taro'\ \ read'\ \ book' \rangle\!\rangle_S$$

"Paradoxical" verbal morphology

Causative and passive verb forms

- *tabe-ru* 'to eat'
- *tabe-sase-ru* 'to cause to eat'
- *tabe-rare-ru* 'to be made to eat'

"Paradoxical" behavior of causative and passive

Morpho-phonologically behaves as a single word

Semantically behaves as if involving complementation

- E.g., adverb scope ambiguity

But this "paradox" is due to traditional notions of constituency

- Kubota 2008 vs GB, LFG, HPSG

General scheme for verbal morphology

Let the dimension E denote eventualities

Verbal roots have types of the general form

$$\langle\!\langle \cdots \to \langle\!\langle \tau \rangle\!\rangle_E \rangle\!\rangle_V$$

Verbal inflections use "multicomposition"

$$(\lambda_V v.\ \langle\!\langle (\lambda_E e.\ f) \cdot v \rangle\!\rangle_S)$$

- v can have **any** arity
- The semantic content f, has access to the **whole** eventuality e
    - So if e is a compound eventuality, f can affect all or part of it

Lexical entries for a few verbal inflections

	Form	=	Semantics
Non-past	$\textit{-(r)u}$	=	$\lambda_V v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e){=}\textsc{Npst}) \cdot v \rangle\!\rangle_S$
Perfect	$\textit{-ta}$	=	$\lambda_V v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e){=}\textsc{Perf}) \cdot v \rangle\!\rangle_S$
Causative	$\textit{-(s)ase-}$	=	$\lambda_V v.\ \langle\!\langle \lambda_{nom}\, n.\ \lambda_{dat}\, d.\ (\lambda_E\, e.\ \langle\!\langle e \wedge \textsc{Cause}(e){=}n \rangle\!\rangle_E) \cdot v \cdot \langle\!\langle d \rangle\!\rangle_{nom} \rangle\!\rangle_V$
Passive	$\textit{-(r)are-}$	=	$\lambda_V v.\ \langle\!\langle \lambda_{nom}\, n.\ \lambda_{dat}\, d.\ (\lambda_E\, e.\ \langle\!\langle e \wedge \textsc{Exper}(e){=}n \rangle\!\rangle_E) \cdot v \cdot \langle\!\langle d \rangle\!\rangle_{nom} \rangle\!\rangle_V$

Semantic analysis for *yonda*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{\textit{yon-}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \qquad \overset{\textit{-da}}{\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S}}{(\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \blacktriangleleft}{\langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E) \rangle\!\rangle_S} \beta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_S} \eta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a\ \wedge\ \textsc{Tense}(n\ reads'\ a) = \textsc{Perf} \rangle\!\rangle_E \rangle\!\rangle_S} \beta}{}$$

Semantic analysis for *yonda*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{yon\text{-}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \qquad \overset{\text{-}da}{\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S}}{(\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \blacktriangleleft}{\langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E) \rangle\!\rangle_S} \beta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_S} \eta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a\ \wedge\ \textsc{Tense}(n\ reads'\ a) = \textsc{Perf} \rangle\!\rangle_E \rangle\!\rangle_S} \beta}{}$$

Semantic analysis for *yonda*

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\dfrac{\overset{yon\text{-}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n \textit{ reads}'\ a \rangle\!\rangle_E \rangle\!\rangle_V}
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{(\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n \textit{ reads}'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \blacktriangleleft
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{\langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n \textit{ reads}'\ a \rangle\!\rangle_E) \rangle\!\rangle_S} \beta
}
{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot \langle\!\langle n \textit{ reads}'\ a \rangle\!\rangle_E \rangle\!\rangle_S} \eta
}
{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n \textit{ reads}'\ a \wedge \textsc{Tense}(n \textit{ reads}'\ a) = \textsc{Perf} \rangle\!\rangle_E \rangle\!\rangle_S} \beta
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Semantic analysis for *yonda*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{yon\text{-}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \qquad \overset{\text{-}da}{\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S}}{(\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \blacktriangleleft}{\langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E) \rangle\!\rangle_S} \beta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_S} \eta}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a\ \wedge\ \textsc{Tense}(n\ reads'\ a) = \textsc{Perf} \rangle\!\rangle_E \rangle\!\rangle_S} \beta}{}$$

Semantic analysis for *yonda*

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        \dfrac{\dfrac{\textit{yon-}}{\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \qquad \dfrac{\textit{-da}}{\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S}}
        {(\lambda_V\, v.\ \langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot v \rangle\!\rangle_S) \cdot \langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_V} \blacktriangleleft
      }
      {\langle\!\langle (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot (\lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a \rangle\!\rangle_E) \rangle\!\rangle_S} \beta
    }
    {\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ (\lambda_E\, e.\ e \wedge \textsc{Tense}(e) = \textsc{Perf}) \cdot \langle\!\langle n\ reads'\ a \rangle\!\rangle_E \rangle\!\rangle_S} \eta
  }
  {\langle\!\langle \lambda_{acc}\, a.\ \lambda_{nom}\, n.\ \langle\!\langle n\ reads'\ a\ \wedge\ \textsc{Tense}(n\ reads'\ a) = \textsc{Perf} \rangle\!\rangle_E \rangle\!\rangle_S} \beta
}{}
$$

The η is a lie!

Conclusion

Associative λ-calculi

- Justifies shorthands in category theory

Chiastic λ-calculi (namely $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi L}$)

- Captures linguistic phenomena
- Type reduction is CR and SN
- Term reduction is WCR

Current work

- Is term reduction SN?
- Can we describe $\Gamma \vdash e : \tau$ more directly?
- What about η?

$\sim$*fin.*

Type reduction is strongly normalizing

Type reduction is Church–Rosser

Type reduction for $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi_L}$ is strongly normalizing

Definition The "length" of a type is the number of constructors

$$
\begin{aligned}
length(T) &= 1 \\
length(\sigma \xrightarrow{A} \tau) &= 1 + length(\sigma) + length(\tau) \\
length(\langle\!\langle \tau \rangle\!\rangle_A) &= 1 + length(\tau) \\
length(\sigma \cdot \tau) &= 1 + length(\sigma) + length(\tau)
\end{aligned}
$$

Lemma Equivalent types have equal length.

Theorem Type reduction diminishes length; i.e.,

$$\forall \tau, \tau'.\ \ \tau \rightsquigarrow \tau' \ \Rightarrow\ length(\tau) > length(\tau')$$

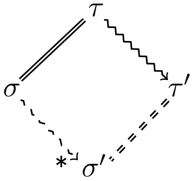
Back

Type reduction is strongly normalizing

Type reduction is Church–Rosser

Type reduction for $\langle\!\langle \lambda_{\rightarrow} \rangle\!\rangle_{\chi L}$ is Church–Rosser

Lemma Type reduction commutes with type equivalence; i.e.,



Theorem Type reduction is weak Church–Rosser.

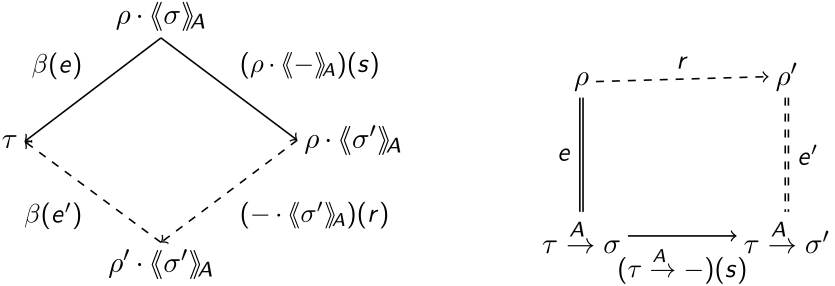
Proof There are no critical pairs. Use the key lemma to resolve potential conflicts between β and itself.

Corollary Type reduction is Church–Rosser

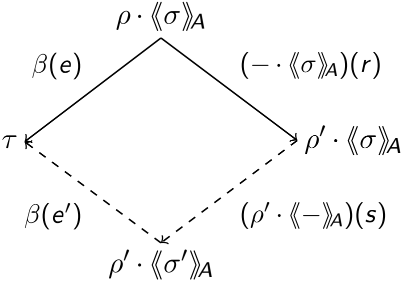
Proof By Newman's Lemma.

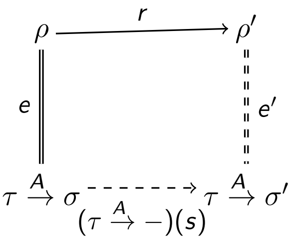
Back

Case 2

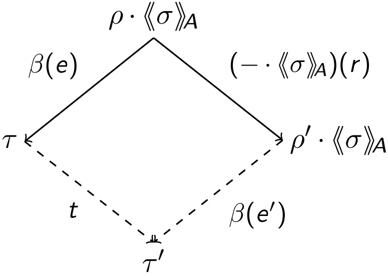

$\rho \cdot \langle\!\langle \sigma \rangle\!\rangle_A$
$\beta(e)$
$(\rho \cdot \langle\!\langle - \rangle\!\rangle_A)(s)$
τ
$\rho \cdot \langle\!\langle \sigma' \rangle\!\rangle_A$
$\beta(e')$
$(- \cdot \langle\!\langle \sigma' \rangle\!\rangle_A)(r)$
$\rho' \cdot \langle\!\langle \sigma' \rangle\!\rangle_A$
ρ
r
ρ'
e
e'
$\tau \xrightarrow{A} \sigma$
$\tau \xrightarrow{A} \sigma'$
$(\tau \xrightarrow{A} -)(s)$

Case 3a

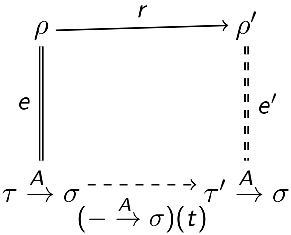

$\rho \cdot \langle\!\langle \sigma \rangle\!\rangle_A$
$\beta(e)$
$(- \cdot \langle\!\langle \sigma \rangle\!\rangle_A)(r)$
τ
$\rho' \cdot \langle\!\langle \sigma \rangle\!\rangle_A$
$\beta(e')$
$(\rho' \cdot \langle\!\langle - \rangle\!\rangle_A)(s)$
$\rho' \cdot \langle\!\langle \sigma' \rangle\!\rangle_A$


r
ρ
ρ'
e
e'
$\tau \xrightarrow{A} \sigma$
$\tau \xrightarrow{A} \sigma'$
$(\tau \xrightarrow{A} -)(s)$

Case 3b



$\rho \cdot \langle\!\langle \sigma \rangle\!\rangle_A$
$\beta(e)$
$(- \cdot \langle\!\langle \sigma \rangle\!\rangle_A)(r)$
τ
$\rho' \cdot \langle\!\langle \sigma \rangle\!\rangle_A$
t
$\beta(e')$
τ'



ρ
r
ρ'
e
e'
$\tau \xrightarrow{A} \sigma$
$\tau' \xrightarrow{A} \sigma$
$(- \xrightarrow{A} \sigma)(t)$