	$\mathscr{S}$	$\oplus$	$\mathbb{0}$	$\otimes$	$\mathbb{1}$	$\otimes$-comm.	$\otimes$-idem.	$\oplus$-idem.	$\overline{(\cdot)}$	$\ominus$	$(\cdot)^*$
natural numbers	$\mathbb{N}$	$+$	0	$*$	1	✓	✗	✗	✗		✗
compactification of $\mathbb{N}$	$\mathbb{N}\cup\{{}^{+}\infty\}$	$+$	0	$*$[1]	1	✓	✗	✗	✗		$(x\equiv 0)\ ?\ 1 : {}^{+}\infty$
non-negative rationals	$\mathbb{Q}_{[0..{}^{+}\infty)}$	$+$	0	$*$	1	✓	✗	✗	✗		✗
non-negative reals	$\mathbb{R}_{[0..{}^{+}\infty)}$	$+$	0	$*$	1	✓	✗	✗	✗		✗
log-domain non-negative reals	$\mathbb{R}_{[{}^{-}\infty..{}^{+}\infty)}$	log sum	${}^{-}\infty$	$+$	0	✓	✗	✗	✗		✗
integers	$\mathbb{Z}$	$+$	0	$*$	1	✓	✗	✗	${}^{-}x$	$x-y$	✗
integers modulo n	$\mathbb{Z}/n\mathbb{Z}$	$+$	0	$*$	1	✓	✗	✗	${}^{-}x$	$x-y$	✗
rationals	$\mathbb{Q}$	$+$	0	$*$	1	✓	✗	✗	${}^{-}x$	$x-y$	✗
reals	$\mathbb{R}$	$+$	0	$*$	1	✓	✗	✗	${}^{-}x$	$x-y$	✗
one-point compactification[2] of $\mathbb{R}$	$\mathbb{R}\cup\{\dot\infty\}$	$+$[3]	0	$*$[1]	1	✓	✗	✗	✗[4]		$1/(1-x)$[5]
extended Viterbi	$\mathbb{R}_{[0..{}^{+}\infty]}$	max	0	$*$[1]	1	✓	✗	✓	✗		$(x\leq 1)\ ?\ 1 : {}^{+}\infty$
Viterbi	$\mathbb{R}_{[0..{}^{+}\infty)}$	max	0	$*$	1	✓	✗	✓	✗		✗
log-domain Viterbi	$\mathbb{R}_{[{}^{-}\infty..{}^{+}\infty)}$	max	${}^{-}\infty$	$+$	0	✓	✗	✓	✗		✗
probabilistic Viterbi	$\mathbb{R}_{[0..1]}$	max	0	$*$	1	✓	✗	✓	✗		1
log-domain probabilistic Viterbi	$\mathbb{R}_{[{}^{-}\infty..0]}$	max	${}^{-}\infty$	$+$	0	✓	✗	✓	✗		0
non-negative tropical	$\mathbb{R}_{[0..{}^{+}\infty]}$	min	${}^{+}\infty$	$+$	0	✓	✗	✓	✗		0
tropical	$\mathbb{R}_{({}^{-}\infty..{}^{+}\infty]}$	min	${}^{+}\infty$	$+$	0	✓	✗	✓	✗		✗
extended tropical	$\mathbb{R}_{[{}^{-}\infty..{}^{+}\infty]}$	min	${}^{+}\infty$	$+$[6]	0	✓	✗	✓	✗		$(x\geq 0)\ ?\ 0 : {}^{-}\infty$
bounded distributive lattice, L	L	$\sqcup$	$\bot$	$\sqcap$	$\top$	✓	✓	✓	✗		$\top$
bounded totally ordered set, X	$X_{[a..b]}$	max	a	min	b	✓	✓	✓	✗		b
binary Boolean algebra	$\{0,1\}$	$\vee$	0	$\wedge$	1	✓	✓	✓	✗		1
subsets of X, as a Boolean algebra	$\mathscr{P}(X)$	$\cup$	$\varnothing$	$\cap$	X	✓	✓	✓	✗		X
binary Boolean ring	$\{0,1\}$	$\veebar$[7]	0	$\wedge$	1	✓	✓	✗	x[7]	$x\veebar y$[7]	✗
subsets of X, as a Boolean ring	$\mathscr{P}(X)$	$\triangle$[8]	$\varnothing$	$\cap$	X	✓	✓	✗	x[8]	$x\triangle y$[8]	✗
regular languages over Σ	$\mathscr{P}(\Sigma^*)$	$\cup$	$\varnothing$	$\cdot$	$\{\varepsilon\}$	✗	✗	✓	✗		x^*
square matrices over semiring R	$R(n,n)$	$+$	$\mathbf{0}_n$	$*$	$\mathbf{I}_n$	✗	✗	✓[9]	$-x$[10]	$x-y$[10]	x^*[11]

[1] Where we define $0 * {}^{+}\infty = 0 = {}^{+}\infty * 0$ or, as appropriate, $0 * \dot\infty = 0 = \dot\infty * 0$.

[2] Note that our use of the Alexandroff compactification is distinct from the real projective line ($\hat{\mathbb{R}}$, $\mathbb{R}P^1$, or $P^1(\mathbb{R})$). The real projective line interpretation of $\mathbb{R}\cup\{\dot\infty\}$ requires leaving the following six expressions undefined: $\dot\infty+\dot\infty$, $\dot\infty-\dot\infty$, $0*\dot\infty$, $\dot\infty*0$, $\dot\infty/\dot\infty$, and $0/0$. However, to properly form a $*$-semiring we require defining all but the last two.

[3] Where we define $\dot\infty+\dot\infty=\dot\infty$.

[4] Beware, this is not a ring! The natural negation operation has that ${}^{-}\dot\infty=\dot\infty$, so negation does not always give the additive inverse. Together with our definition of addition this means: $\dot\infty-\dot\infty=\dot\infty+{}^{-}\dot\infty=\dot\infty+\dot\infty=\dot\infty$.

[5] Except that we define $1^*=\dot\infty$ and $\dot\infty^*=\dot\infty$.

[6] Where we define ${}^{+}\infty+{}^{-}\infty={}^{+}\infty$.

[7] That is, exclusive or: $x\veebar y=(x\vee y)\wedge\neg(x\wedge y)=(x\wedge\neg y)\vee(y\wedge\neg x)=\neg(x\Leftrightarrow y)$. Note that $x\veebar x=0$. Thus, do not confuse the Boolean ring's negation (the identity function) with the Boolean algebra's complementation (i.e., $\neg y$).

[8] That is, symmetric difference: $x\triangle y=(x\cup y)\setminus(x\cap y)=(x\setminus y)\cup(y\setminus x)$. Note that $x\triangle x=\varnothing$. Thus, do not confuse the Boolean ring's negation (the identity function) with the Boolean algebra's complementation (i.e., $\overline{y}$).

[9] Provided R forms a dioid (i.e. $\oplus$-idempotent semiring).

[10] Provided R forms a ring.

[11] Provided R forms a $*$-semiring (i.e., closed semiring). An efficient implementation is the famous Gauss–Jordan–Floyd–Warshall–McNaughton–Yamada algorithm. Beware that the notation for asteration should not be confused with the antiautomorphic involution (e.g., conjugate transpose) of $\star$-rings (i.e., ring with involution).